The Unreasonable Effectiveness of Mathematics

'The Unreasonable Effectiveness of Mathematics in the Natural Sciences' – this is the title of a widely cited paper [1] by the Nobel winning Hungarian Physicist Eugene P. Wigner, where he famously expresses his sense of puzzled wonder at how mathematics comes to possess such unfathomable effectiveness in describing the mechanisms of the natural world. In the words of the great mathematical physicist, “[The first point is that] the enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious and [that] there is no rational explanation for it”.

The point that Wigner tries to make here relates to a really deep question that has baffled mathematicians and natural scientists alike for centuries, and raises fundamental issues concerning the very nature and identity of mathematics. Mathematics is, in a manner of speaking, a *mind* game, while the natural sciences are empirical in nature, involving the formation of concepts from perceptual experience. To an extent, mathematics also appears to have empirical roots, as in the formation of the concepts relating to the integers and to the relatively elementary mathematical notions. However, the deeper concepts in mathematics have no discernible link with the perceptual reality, and yet they are uncannily effective in describing the workings of the natural world. Wigner cites the examples of the theory of relativity and quantum theory in support of his point where the concepts of non-Euclidean four dimensional space and of the infinite dimensional Hilbert space constitute the mathematical settings in which these two theories have been formulated, with enormous success in explaining natural phenomena.

In the same vein as Wigner, many great names in science have expressed their bemused wonder at this contrariness that mathematics seems to engender. For instance, Steven Weinberg, another Nobel winning physicist, has had this to say [2]: “It is very strange that mathematicians are led by their sense of mathematical beauty to develop formal structures that physicists only later find useful, even where the mathematician had no such goal in mind. [. . .] Physicists generally find the ability of mathematicians to anticipate the mathematics needed in the theories of physics quite uncanny. It is as if Neil Armstrong in 1969 when he first set foot on the surface of the moon had found in the lunar dust the footsteps of Jules Verne.”

The same puzzlement has been expressed by many others, including the great Einstein who wondered [3]: “How can it be that mathematics, being after all a product of human thought which is independent of experience, is so admirably appropriate to the objects of reality? Is human reason, then, without experience, merely by taking thought, able to fathom the properties of real things?"

All this incomprehension brings up a number of focal issues in the philosophy of mathematics. Many of these are traditionally formulated in terms of opposing trends of thought such as the one relating to the *realist* and the *anti-realist* points of view. Briefly speaking, realism looks at mathematical concepts as objectively existing entities, though in a world different from the one we live in. When the human mind comprehends any of these objects, it just 'discovers' it since it was already 'there', independently of the comprehending mind. The anti-realist, on the other hand, thinks that the mind 'invents' mathematical notions in the process of a creative game. Incidentally, Wigner seems to have adopted an anti-realist position in his paper mentioned above. And interestingly, both the two views come up with the same problem of explaining the applicability of mathematical notions to the physical world - the world of our experience.

Can it be that the perceived contradictions expressed by the likes of Wigner, Weinberg, and Einstein are the result of an attempt to look at mathematics and the natural sciences in too logical and abstract terms? After all, both mathematics and the sciences are products of the workings of the human mind, and they must have common links in the same human thought process that takes in data from the physical world and then goes on to create 'worlds' of its own. This is not to deny that the relationship between mathematics and the sciences does pose deep and complex questions that have to be addressed so as to enrich our idea as to what really goes on when the human mind develops mathematical notions that come to possess a great degree of relevance to the sciences. What seems to have been overlooked is that the answer to these questions is not to be sought through a discourse spirited away from the concrete workings of the human mind.

There are dangers in making a question too deep, too profound, and too abstract - more deep and more abstract than the context in which it arises. An analogy that comes to mind is the way the 'riddle of induction' has for centuries baffled the best philosophical minds, where no solution to this riddle could ever be found on grounds of 'pure logic'. One will not be far from the truth in stating that 'pure' deductive logic has traditionally been assumed to encapsulate the way the human mind engages in activities relating to reasoning. This was formalized by the great logician and philosopher of mathematics Gottlob Frege in his attempts to reduce mathematics, or a sizable chunk of it, to logic (see, for instance, [4]). But the reality of the situation is that 'induction' is also a major component of human reasoning, and it is a contradiction in terms to try to explain induction in a purely 'logical' manner, to explain one aspect of human reasoning by the other. It is a reality that induction cannot be formalized in the same way as deductive logic, and new light has been thrown on the riddle of induction by researches in cognitive science that try to grasp the process of induction in the actual working of the human mind, which proceeds through series of *half-baked and tentative* notions that end up in relatively well defined concepts amenable to logical analysis.

Incidentally, this same 'riddle' of induction seems to be one of the factors at work in giving mathematics its distinctive identity as compared to the natural sciences, since induction appears to work in a distinctive manner in the former in comparison to the latter.

With all this, however, there is no question of the 'mystery' going away – the mystery of the effectiveness of mathematics in the natural sciences that has troubled Wigner, Weinberg, and Einstein, and continues to haunt other great minds of our time.......... some mysteries just *refuse* to go away. These are mysteries generated when the human mind tries to comprehend the infinitude of the world out there, or the unfathomable depths of the mind itself.

[1] E. P. Wigner, *The Unreasonable Effectiveness of Mathematics in the Natural Sciences,* Richard Courant Lecture in Mathematical Sciences, New York University, May 11, 1959; Comm Pure Appl. Math., vol. 13, pp 1-14 (1960).

[2] Steven Weinberg, *Dreams of a Final theory,* Vintage Books, N.Y. (1993); quoted in Mark Colyvan, *An Introduction to the Philosophy of Mathematics,* Cambridge University Press, cambridge (2012).

[3] Albert Einstein, *Sidelights on Relativity,* Dover, N.Y. (1983); quoted in [4].

[4] A. George and D. J. Velleman, *Philosophies of Mathematics,* Blackwell Publishers Inc., Massachusetts (2002).

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